Experiments on the stability of hydromagnetic Couette flow

By R. J. DONNELLY AND D. R. CALDWELL

Institute for the Study of Metals and Department of Physics,
The University of Chicago, Chicago, Illinois

(Received 1 November 1963)

The viscometer investigations of Donnelly & Ozima (1962) on the stability of flow between rotating cylinders in the presence of a magnetic field have been extended by making careful torque measurements in the subcritical range using improved mercury. The results show that a small difference in stability can be observed with conducting cylinders as compared with insulating cylinders. The results also suggest that a heretofore unnoticed instability begins well before the instability studied earlier. This relatively weak instability accounts for some of the anomalous effective viscosity observed by Donnelly & Ozima.

1. Introduction

The stability of hydromagnetic Couette flow was investigated by Donnelly & Ozima (1962) using a rotating cylinder viscometer. The results with insulating cylinders agreed very well with theoretical calculations (Chandrasekhar 1961) for the onset of the axisymmetric instability. Although the calculations indicate an easily detectable change in critical speed when the cylinders are perfectly conducting, a set of stainless-steel cylinders gave results which could not be distinguished from those for insulating cylinders. A further problem was that at subcritical speeds the effective viscosity was found to increase with both the magnetic field and the rate of rotation, whereas it should be constant for a truly laminar flow. We have undertaken a new series of measurements with the same viscometer in order to try to resolve these two problems. These experiments show that with sufficiently pure mercury a small difference in stability can be detected with conducting cylinders. The results also suggest that the increase in effective viscosity is due to the onset of a relatively weak instability at low speeds of rotation of the inner cylinder. The onset was clearly defined over only a limited range of magnetic field strength.

2. The problem of the anomalous effective viscosity

The effective viscosity in a Couette viscometer is proportional to the product of the deflexion of the outer cylinder ϕ and the period of rotation of the inner cylinder $P\,(=2\pi/\Omega)$, where Ω is the rate of rotation in rad/sec (see Donnelly & Ozima 1962). The first suggestion for the origin of the extra effective viscosity in the subcritical range was that an extra torque was developed due to circulating currents induced by the rotation of the inner (metal) cylinder. Substitution of insulating cylinders did not materially reduce the effect, eliminating that possibility.

A further possibility is that the mercury was somehow contaminated. This has been extensively studied by Caldwell (1963) using an oscillating-disk viscometer in a magnetic field. His results showed that for an insulating-disk the results for the damping could be accounted for exactly by a straightforward hydromagnetic calculation. On the other hand, with a conducting disk the results were much different, but this difference only appeared if extensive precautions were taken to clean the mercury and the surfaces in contact with it. Details are contained in his publications, but the results suggested that the fact that no difference in stability was observed between insulating and conducting cylinders might be connected with the state of the mercury and the cylinder surfaces.

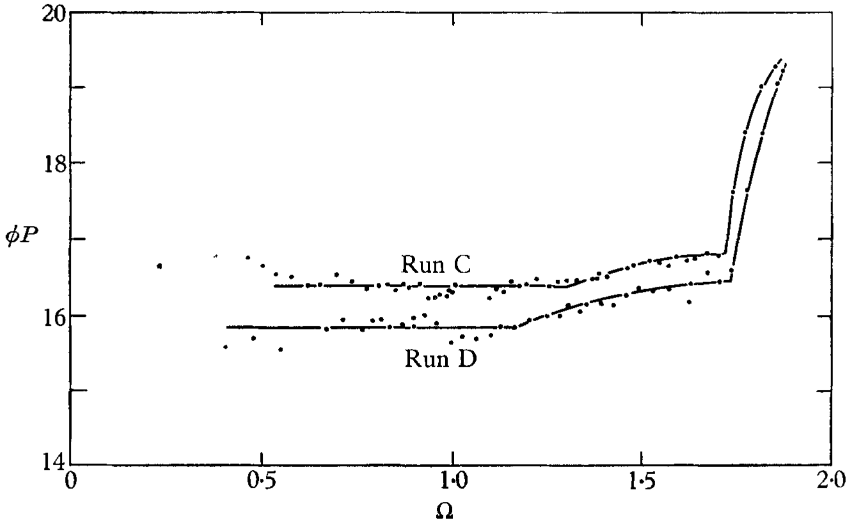

FIGURE 1. Variation of ϕP, which is proportional to the effective viscosity, with the rate of rotation of the inner cylinder, Ω. The cylinders were stainless steel with radii $R_1 = 1{\cdot}8$, $R_2 = 2{\cdot}0$ cm. Run C was taken with mercury cleaned according to the procedures of Donnelly & Ozima. Run D was taken with cleaner mercury.

A quantity of mercury was specially prepared for this experiment using the procedures outlined by Caldwell. Results obtained on 29 July 1963 with stainless-steel cylinders of radii $R_1 = 1{\cdot}8$, $R_2 = 2{\cdot}0$ cm and at a field of 3000 G are shown in figure 1. Run C was taken with mercury treated only according to the procedures used by Donnelly & Ozima. Run D was taken with the better mercury. Two effects may be seen: first, the effective viscosity, ϕP, is lower with cleaner mercury, and second, the transition to instability occurs at a slightly higher angular velocity with the cleaner mercury: $\Omega_c = 1{\cdot}725$ rad/sec for run C and $\Omega_c = 1{\cdot}740$ rad/sec for run D. Thus it appears reasonable to conclude that a definite increase in stability occurs with conducting cylinders, but that it is normally masked by surface and mercury contamination. Quantitative assessment of the results is obviously very difficult as we have no quantitative measure of the nature of our mercury or the state of the surfaces of the cylinders. However, calculations by Roberts (to be published this year) appropriate to this experiment indicate that a considerably greater difference should be observable.

Referring again to figure 1, we see that there are two regions of interest in the subcritical range: a region with ϕP independent of Ω and a region where ϕP is slowly increasing. [In assessing the results of these plots, one should recall that the accuracy falls off at low speeds due to the decrease in torque.] The results suggest that there may be a relatively weak instability which begins just above $\Omega = 1{\cdot}0$ rad/sec, and that it is this earlier instability which accounts for at least part of the anomalous effective viscosity noticed by Donnelly & Ozima. The remaining variation in effective viscosity is considered to be due to impurities in the mercury and surfaces in contact with it, misalignment and eccentricity of the cylinders, as well as end effects due to the increasing wavelength of disturbances at high fields. Comparison of runs C and D shows that the speed at which this first instability begins is influenced by the purity of the mercury. This means that the results to be presented in the following section are subject to uncertainties which at the present state of our technical knowledge could not be overcome. The experiments were carried out with as pure mercury as we could prepare with the expectation that the results would be qualitatively correct. A further difficulty is the size of the effect. Even in run D, the total rise of ϕP between the two critical speeds is only 4 %, not much above the scatter of results at lower speeds.

3. Experiments to determine the onset of instability

A systematic set of measurements were taken to explore further the nature of the first instability. Results with stainless-steel cylinders having radii $R_1 = 1{\cdot}8$, $R_2 = 2{\cdot}0$ cm are shown in figure 2. The runs at 3000, 4000, 4500, 5000 and 6000 G show quite definite evidence for two critical velocities. At 7000 G, the evidence for a first break is questionable, and at higher fields there is no evidence of the first break in the curve. Indeed at 10,000 G it is difficult to locate the break in the curve for even the axisymmetric transition.

The rise in effective viscosity beyond the first transition becomes generally more pronounced as the field is increased. Below 3000 G, there is insufficient increase in ϕP to detect the first critical velocity. While the second transition varies rapidly with magnetic field (in accordance with Chandrasekhar's calculations), the first transition is relatively constant at about 1·0 rad/sec.

After these observations were completed, the inner cylinder was replaced by one of stainless steel having a radius $R_1 = 1{\cdot}9$ cm, leaving a 1 mm gap. Detailed torque measurements at a number of different magnetic fields are shown in figure 3. The pattern of results is qualitatively similar to those for the 2 mm gap. A lower critical velocity can be identified at 2000, 3500, 4000, 4500 and 5000 G. This critical angular velocity is in the range 1·0–1·5 rad/sec. Above 5000 G, there is an irregularity in the effective viscosity curves at about this speed, but not of regular form.

Finally, the cylinders were replaced with insulating cylinders of radii $R_1 = 1{\cdot}9$, $R_2 = 2{\cdot}0$ cm. The results, shown in figure 4, are in good agreement with those shown in figure 3 with critical velocities identified at 3000, 4000 and 5000 G.

4. Discussion

The results of the preceding section suggest the presence of a new instability which begins before the axisymmetric one. There is always a possibility that this apparent break in the ϕP curve is due to some extraneous effect, such as the finite length of the cylinders. However, changing from a 2 mm gap to a 1 mm gap doubles the effective length of the annulus in wavelength units, and the break in the ϕP curve is still clearly evident. The new instability is relatively weak compared to the other: the rise in ϕP amounts to only a few percent, increasing somewhat with increasing magnetic field. The onset of instability could be clearly located only between 3000 and 7000 G, and in this range it does not vary strongly

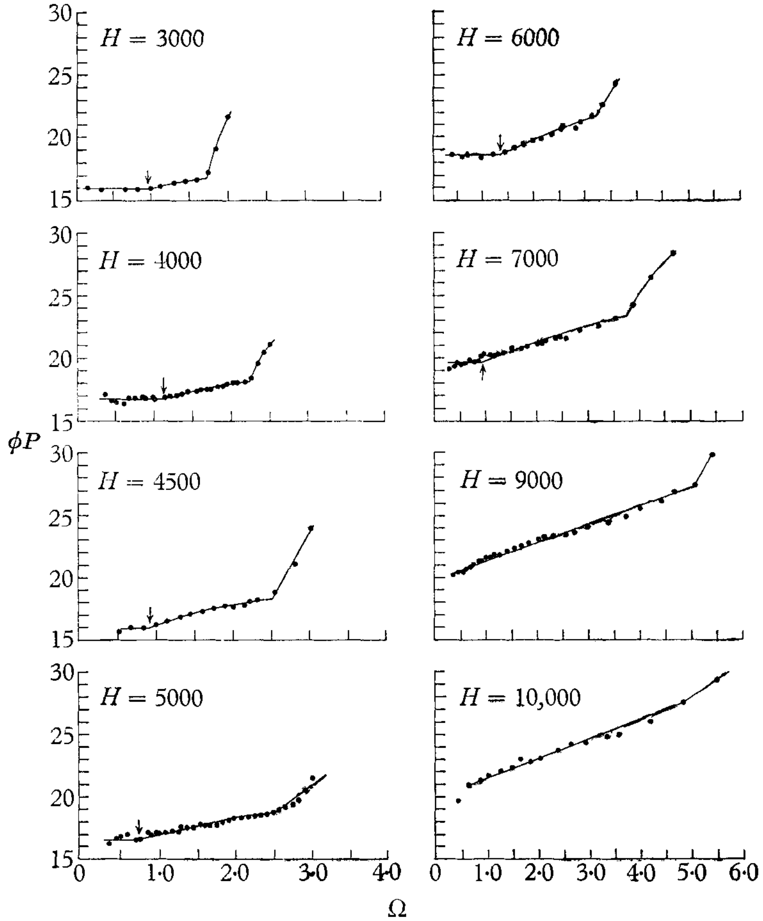

FIGURE 2. Effective viscosity measurements taken with stainless-steel cylinders of radii $R_1 = 1{\cdot}8$ cm, $R_2 = 2{\cdot}0$ cm at a number of different magnetic field strengths, H, given in Gauss. The arrows indicate the estimated angular velocity for the onset of instability.

with magnetic field. For example, defining the Taylor number T and Q as in Donnelly & Ozima (1962), we find that with a 1 mm gap a critical velocity of 1·5 rad/sec at 6000 G and 23·5 °C corresponds to $T = 2900$ and $Q = 230$, and the same data with a 2 mm gap corresponds to $T = 22{,}000$ and $Q = 970$. This is to be contrasted with the symmetric mode which has $T = 2887$ at $Q = 16{\cdot}70$ and $T = 20{,}210$ at $Q = 179{\cdot}6$. Changing from conducting to insulating cylinders does not noticeably change the results, whereas the purity of the mercury does.

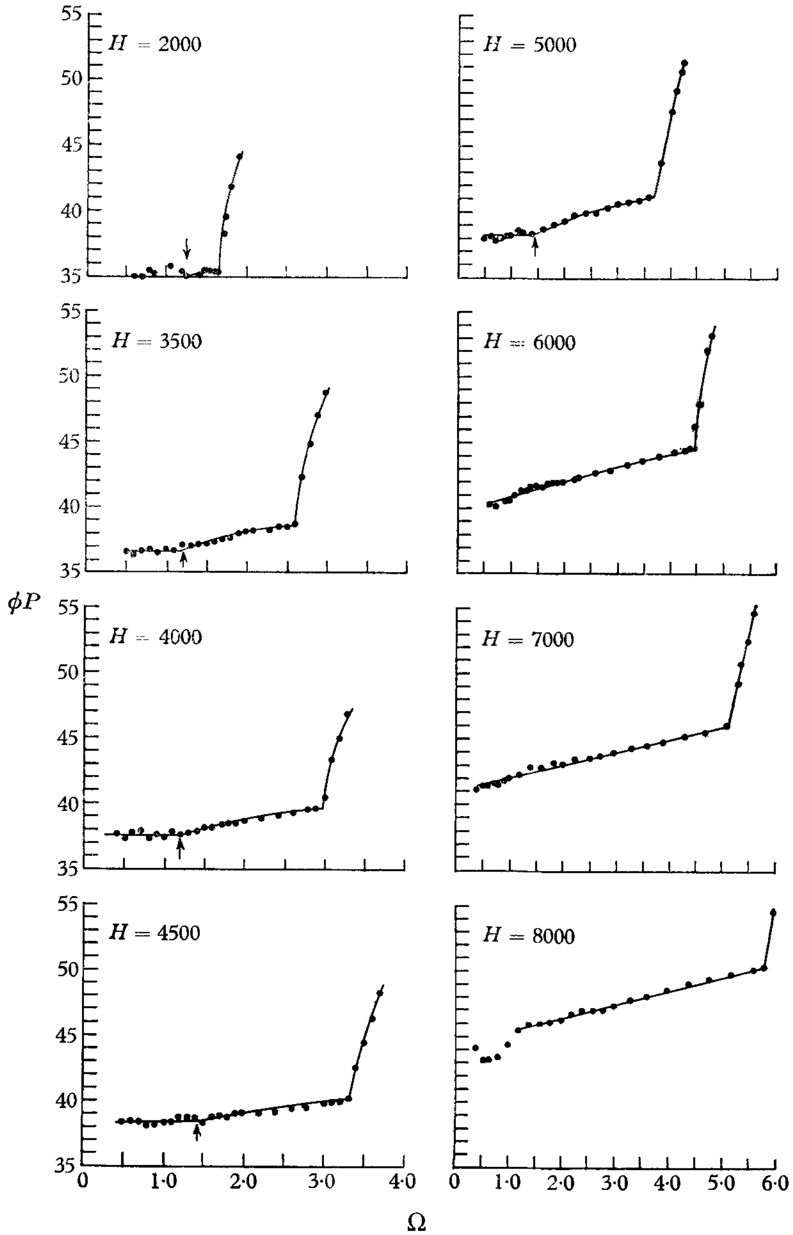

FIGURE 3. Effective viscosity measurements taken with stainless-steel cylinders of radii $R_1 = 1{\cdot}9$ cm, $R_2 = 2{\cdot}0$ cm.

It is tempting to speculate on the nature of this newly discovered mode. These experiments were undertaken after Prof. P. H. Roberts suggested the possibility that a magnetic field might have a smaller inhibiting effect on a non-symmetric mode (such as has been discussed by Di Prima 1961) than on the axisymmetric mode, so that beyond a certain magnetic field the lowest mode of instability would be a non-symmetric one. Concurrently, Prof. Roberts carried out extensive theoretical investigations of the problem which are reported elsewhere (Roberts 1964). The results of these calculations have not disproved his hypothesis, but on several counts made it seem unlikely. We are left with the situation still less than satisfactory: the experiments have proved to be limited in the precision which can be achieved due to the problems of preparation of mercury, and the theory has proved to entail severe difficulties and uncertainties which are still to be resolved.

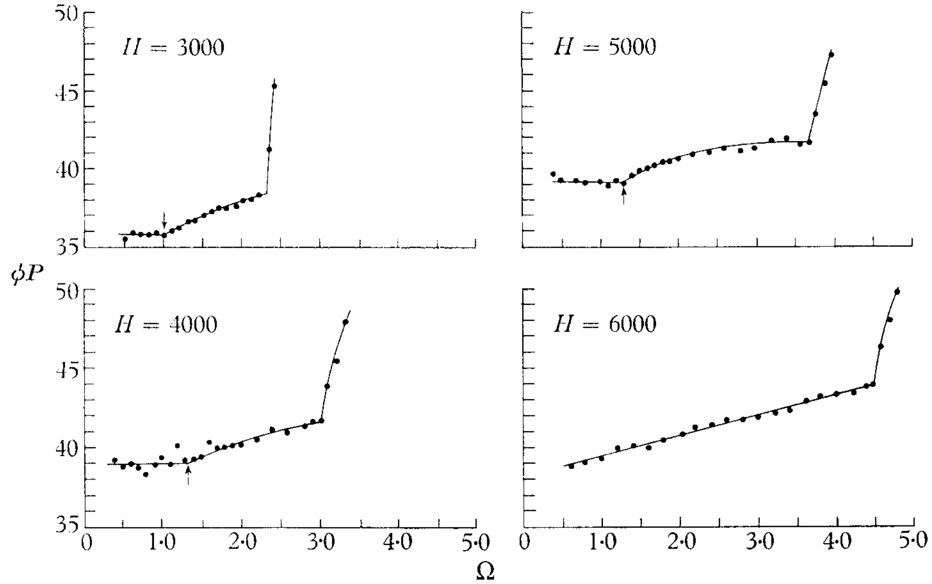

FIGURE 4. Effective viscosity measurements taken with insulating cylinders of radii $R_1 = 1{\cdot}9$ cm, $R_2 = 2{\cdot}0$ cm.

The authors are grateful to Prof. P. H. Roberts for his suggestion which reopened the examination of this problem. We are also grateful to Prof. S. Chandrasekhar for his encouragement and comments on the experiments. We are indebted to Mr R. W. Koster for assistance in the preparation of the mercury and in taking the measurements. This research has been supported by the Office of Naval Research under a contract Nonr-2121(20) with the University of Chicago and by the National Science Foundation under a grant G-23068 for the Physics of Fluids. Research activities of the Institute for the Study of Metals are supported by general grants from the U.S. Atomic Energy Commission and the Advanced Research Projects Agency. During the period of this research, R. J. Donnelly held an Alfred P. Sloan Research Fellowship.

REFERENCES

Caldwell, D. R. 1963 Oscillating boundary layer in magnetohydrodynamics. I. Non-conducting boundary. II. Conducting boundary. *Phys. Fluids* (in Press).

Chandrasekhar, S. 1961 *Hydrodynamic and Hydromagnetic Stability*. Oxford: Clarendon Press.

Di Prima, R. C. 1961 *Phys. Fluids*, **4**, 751.

Donnelly, R. J. & Ozima, M. 1962 *Proc. Roy. Soc.* A, **266**, 272.